CBSE and ISC BOARD

Class 12th

DEFINITE INTEGRALS

IMPORTANT Questions

Evaluate the following definite integrals:

$$\int_0^1 (2x^3+3)^2\,dx$$

$$I = \int_0^1 (2x^3+3)^2\,dx$$

$$= \int_0^1 (4x^6+9+12x^3)\,dx$$

$$= 4\int_0^1 x^6\,dx + 9\int_0^1 dx + 12\int_0^1 x^3\,dx$$

$$= 4\left[\frac{x^7}{7}\right]_0^1 + 9[x]_0^1 + 12\left[\frac{x^4}{4}\right]_0^1$$

$$4\left[\frac{(1)^7}{7} - \frac{0}{7}\right] + 9[1-0] + 12\left[\frac{1}{4}-0\right]$$

$$\frac{4}{7} + 9 + \frac{12}{4}$$

$$\frac{4}{7} + 9 + 3$$

$$\frac{4}{7} + 12$$

$$\frac{4+84}{7}$$

$$= \frac{88}{7} \text{ Ans}$$

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If $\int_0^a \frac{1}{4+x^2}\,dx = \frac{\pi}{8}$ Find the value of a

$$\int_0^a \frac{1}{(2)^2+x^2}\,dx = \frac{\pi}{8}$$

$$\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^a = \frac{\pi}{8}$$

$$\frac{1}{2}\left[\tan^{-1}\frac{a}{2} - \tan^{-1}\frac{0}{2}\right] = \frac{\pi}{8}$$

$$\frac{1}{2}\left[\tan^{-1}\frac{a}{2} - \tan^{-1}0\right] = \frac{\pi}{8}$$

$$\left[\tan^{-1}\frac{a}{2}\right] = \frac{\pi}{4}$$

$$\frac{a}{2} = \tan\frac{\pi}{4}$$

$$\frac{a}{2} = 1$$

$$\boxed{a = 2}$$

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$$\int_0^{\pi/2} \cos^2 x \, dx$$

$\sin 180^\circ = \sin(90+90)$

$\cos 90$

$\cos 2x = 2\cos^2 x - 1$

$$\frac{\cos 2x + 1}{2} = \cos^2 x$$

$$I = \int_0^{\pi/2} \cos^2 x \, dx$$

$$= \int_0^{\pi/2} \left(\frac{\cos 2x + 1}{2}\right) dx$$

$$= \frac{1}{2}\left[\int_0^{\pi/2} \cos 2x \, dx + \int_0^{\pi/2} 1 \, dx\right]$$

$$\frac{1}{2}\left[\left[\frac{\sin 2x}{2}\right]_0^{\pi/2} + [x]_0^{\pi/2}\right]$$

$$\frac{1}{2}\left[\frac{1}{2}\left[\sin 2x \frac{\pi}{2} - \sin 0\right] + \left[\frac{\pi}{2} - 0\right]\right]$$

$$\frac{1}{2}\left[[0] + \frac{\pi}{2}\right] = \frac{\pi}{4}$$

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$$\int_{\pi/4}^{\pi/2} \sqrt{1-\sin 2x}\, dx$$

$$I = \int_{\pi/4}^{\pi/2} \sqrt{1-\sin 2x}\, dx$$

$$= \int_{\pi/4}^{\pi/2} \sqrt{(\sin^2 x + \cos^2 x - 2\sin x \cos x)}\, dx$$

$$= \int_{\pi/4}^{\pi/2} \sqrt{(\sin x - \cos x)^2}\, dx$$

$$I = \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\, dx$$

$$I = \int_{\pi/4}^{\pi/2} \sin x\, dx - \int_{\pi/4}^{\pi/2} \cos x\, dx$$

$$= -\left[\cos x\right]_{\pi/4}^{\pi/2} - \left[\sin x\right]_{\pi/4}^{\pi/2}$$

$$= -\left[\cos\frac{\pi}{2} - \cos\frac{\pi}{4}\right] - \left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right]$$

$$= -\left[0 - \frac{1}{\sqrt{2}}\right] - \left[1 - \frac{1}{\sqrt{2}}\right] = \frac{2}{\sqrt{2}}$$ Ans

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$$\int_0^{\pi/2} e^x(\sin x - \cos x)\,dx$$

$$I = \int_0^{\pi/2} e^x(\sin x - \cos x)\,dx$$

$$= \int_0^{\pi/2} e^x \sin x\,dx - \int_0^{\pi/2} e^x(\cos x)\,dx$$

$$\left[e^x \int \sin x\,dx - \int\left(\frac{d}{dx}e^x \int \sin x\,dx\right)dx\right]_0^{\pi/2} - \int_0^{\pi/2} e^x \cos x\,dx$$

$$= \left[-e^x \cos x\right]_0^{\pi/2} + \int_0^{\pi/2} e^x \cos x\,dx - \int_0^{\pi/2} e^x \cos x\,dx = -\left[e^x \cos x\right]_0^{\pi/2}$$

$$= -\left[e^{\pi/2}\cos\frac{\pi}{2} - \cos 0\, e^0\right]$$

$$= -\left[e^{\pi/2}\,0 - 1\times 1\right] = -1[-1]$$

$$= 1 \text{ Ans}$$

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$$\int_0^1 \frac{xe^x}{(x+1)^2}\,dx$$

$$I = \int_0^1 \frac{xe^x}{(x+1)^2}\,dx$$

$$= \int_0^1 \frac{(x+1-1)\,e^x}{(x+1)^2}\,dx$$

$$= \int_0^1 \frac{x+1}{(x+1)^2} e^x\,dx - \int_0^1 \frac{1}{(x+1)^2} e^x\,dx$$

$$= \int_0^1 \frac{e^x}{(x+1)}\,dx - \int_0^1 \frac{1}{(x+1)^2} e^x\,dx$$

$$\left[\frac{1}{(x+1)}\int e^x\,dx - \int\left(\frac{d}{dx}\frac{1}{(x+1)}\int e^x\,dx\right)dx\right]_0^1 - \int_0^1 \frac{1}{(x+1)^2} e^x\,dx$$

$$\left[\frac{e^x}{(x+1)} + \int \frac{1}{(x+1)^2} e^x\,dx - \int_0^1 \frac{1}{(x+1)^2} e^x\,dx\right]_0^1$$

$$\left[\frac{e^x}{(x+1)}\right]_0^1$$

$$\left[\frac{e^1}{1+1} - \frac{e^0}{0+1}\right] = \frac{e}{2} - 1 \quad \text{Ans}$$

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$$\int_0^1 \frac{\tan^{-1} x}{1+x^2}\,dx$$

$$I = \int_0^1 \frac{\tan^{-1} x}{1+x^2}\,dx$$

$$= \int_0^{\pi/4} t\,dt$$

$$= \left[\frac{t^2}{2}\right]_0^{\pi/4} = \frac{1}{2}\left[t^2\right]_0^{\pi/4} = \frac{1}{2}\left[\left(\frac{\pi}{4}\right)^2 - 0\right]$$

$$= \frac{1}{2}\left[\frac{\pi^2}{16}\right] = \frac{\pi^2}{32}$$

$\tan^{-1} x = t$

d.w.r.to

$$\frac{1}{1+x^2}\,dx = dt$$

$x = 1$

$\tan^{-1}(1) = t$

$$\frac{\pi}{4} = t$$

$x = 0$

$\tan^{-1} 0 = t$

$t = 0$

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$$\int_0^{\pi/4} 2\tan^3 x\, dx$$

$$I = \int_0^{\pi/4} 2\tan^3 x\, dx$$

$$= 2\int_0^{\pi/4} (\tan^2 x \tan x)\, dx$$

$$= 2\int_0^{\pi/4} (\sec^2 x - 1)\tan x\, dx$$

$$= 2\left[\int_0^{\pi/4} \sec^2 x \tan x\, dx - \int_0^{\pi/4} \tan x\, dx\right.$$

Put $\tan x = t$

$x = \frac{\pi}{4}$

$1 = t$

$x = 0$

$0 = t$

$\sec^2 x\, dx = dt$

$$2\left[\int_0^1 t\, dt - \left[\log(\sec x)\right]_0^{\pi/4}\right]$$

$$2\left[\left[\frac{t^2}{2}\right]_0^1 - \left[\log\left(\sec\frac{\pi}{4}\right) - \log\sec 0\right]\right.$$

H.W

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$$\int_0^1 \frac{dx}{e^x + e^{-x}}$$

$$I = \int_0^1 \frac{dx}{e^x + \frac{1}{e^x}}$$

$$= \int_0^1 \frac{e^x\,dx}{e^x \cdot e^x + 1}$$

$$= \int_0^1 \frac{e^x}{(e^x)^2 + 1}\,dx$$

$$= \int_1^e \frac{1}{t^2+1}\,dt$$

$$\left[\tan^{-1} t\right]_1^e$$

$$\left[\tan^{-1} e - \tan^{-1} 1\right]$$

$$\left[\tan^{-1} e - \tan^{-1} \tan\frac{\pi}{4}\right]$$

$$\left[\tan^{-1} e - \frac{\pi}{4}\right]$$

but $e^x = t$

$$\boxed{e^x\,dx = dt}$$

$x = 1$

$e^1 = t$

$\boxed{t = e}$

$x = 0$

$e^0 = t$

$\boxed{t = 1}$

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$$\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\, dx$$

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$$\int_0^{\sqrt[3]{\pi^2}} \sqrt{x}\cos^2\left(x^{3/2}\right) dx$$

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$$\int_0^{1/2} \frac{\sin^{-1} x}{(1-x^2)^{3/2}}\, dx$$

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Properties of Definite Integrals

$$\int_a^b f(x)dx = \int_a^b f(t)dt$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$\int_a^b f(x)dx = \int_a^c f(x)\,dx + \int_c^b f(x)dx$, where a<c<b

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$\boxed{\int_0^a f(x)dx = \int_0^a f(a-x)dx}$$

$$\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$$

$$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & if\ f(2a-x) = f(x) \\ 0 & if\ f(2a-x) = -f(x) \end{cases}$$

$$\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if\ f(x) is\ an\ even\ function \\ 0 & if\ f(x) is\ an\ odd\ function \end{cases}$$

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$$\int_0^{\pi/4} \log(1+\tan x)\,dx$$

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